1. Two beams, A and B whose photon energies are 3.3 eV and 11.3 eV respectively, illuminate a metallic surface (work function 2.3 eV) successively. The ratio of maximum speed of electrons emitted due to beam A to that due to beam B is: (2024)

(A) 3

(B) 9

(C) 1/3

(D) 1/9

Ans. (C) 1/3

2. The waves associated with a moving electron and a moving proton have the same wavelength λ . It implies that they have the same: (2024)

(A) momentum

(B) angular momentum

(C) speed

(D) energy

Ans. (A) momentum

3. Assertion (A) and Reason (R) type questions. Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (2024)

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Assertion (A): In photoelectric effect, the kinetic energy of the emitted photoelectrons increases with increase in the intensity of the incident light.

Reason (R): Photoelectric current depends on the wavelength of the incident light.

Ans. (D) Assertion (A) is false and Reason (R) is also false.





4. Consider a neutron (mass m) of kinetic energy E and a photon of the same energy. Let λ_n and λ_p be the de Broglie wavelength of neutron and the wavelength of photon respectively. Obtain an expression for λ_n/λ_p . (2024)

Ans. Obtaining an expression for λ_n/λ_p

$$E = \frac{hc}{\lambda p} \implies \lambda p = \frac{hc}{E}$$
$$\lambda n = \frac{h}{p} = \frac{h}{\sqrt{(2mE)}}$$
$$\frac{\lambda n}{\lambda p} = \frac{h}{\sqrt{(2mE)}} \times \frac{E}{hc}$$
$$\frac{\lambda n}{\lambda p} = \sqrt{\left(\frac{E}{2mc^2}\right)}$$

5. (i) A ray of light passes through a triangular prism. Show graphically, how the angle of deviation varies with the angle of incidence ? Hence define the angle of minimum deviation.

(ii) A ray of light is incident normally on a refracting face of a prism of prism angle A and suffers a deviation of angle δ . Prove that the refractive index n of the material of the prism is given by sin A sin (A + δ)/sin A.

(iii) The refractive index of the material of a prism is $\sqrt{2}$. If the refracting angle of the prism is 60°, find the

CLICK HERE

>>

(1) Angle of minimum deviation, and

(2) Angle of incidence. (2024)

Ans.

(i) Graph showing variation of angle of deviation with angle of incidence

Defining angle of minimum deviation

(ii) Proof of refractive index $n = \frac{\sin(A + \delta)}{\sin A}$

- (iii) (1) Finding angle of minimum deviation
 - (2) Angle of Incidence





Minimum deviation angle is defined as the angle at which angle of incidence is equal to the angle of emergence.

Alternatively

At minimum deviation refracted ray inside the prism becomes parallel to the base of the prism.

(ii)



At the face XZ :- $\mu \sin i = 1 \times \sin r$ ----- (1) $r = i + \delta$ [from diagram] ----- (2) In ΔXMN ; A+(90-i) + 90 = 180

$$\Rightarrow A = i$$

Putting eq. (3) & (2) in eq. (1)
 $\mu \sin A = \sin (A + \delta)$
 $\mu = \frac{\sin (A + \delta)}{\sin A}$

(iii)

(1)
$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$
$$\sqrt{2} = \frac{\sin\left(\frac{60+\delta_m}{2}\right)}{\sin 30^{\circ}}$$
$$\Rightarrow \sin\left(\frac{60+\delta_m}{2}\right) = \frac{1}{\sqrt{2}} = \sin 45^{\circ}$$
$$\frac{60+\delta_m}{2} = 45^{\circ} \Rightarrow \delta_m = 30^{\circ}$$
(2)
$$i = \frac{A+\delta_m}{2}$$
$$\Rightarrow i = \frac{60+30}{2}$$

 $i = 45^{\circ}$

6. (i) State Huygens principle. A plane wave is incident at an angle i on a reflecting surface. Construct the corresponding reflected wavefront. Using this diagram, prove that the angle of reflection is equal to the angle of incidence.

----- (3)

(ii) What are the coherent sources of light ? Can two independent sodium lamps act like coherent sources ? Explain.

(iii) A beam of light consisting of a known wavelength 520 nm and an unknown wavelength λ , used in Young's double slit experiment produces two interference patterns such that the fourth bright fringe of unknown wavelength coincides with the fifth bright fringe of known wavelength. Find the value of λ . (2024)

Ans.

(i) Statement of Huygens' Principle

Construction of reflected wave front

Proof of angle of reflection is equal to angle of incidence

(ii) Definition of coherent sources

Explanation

(iii) Finding the unknown wavelength





(i) Each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the spread of the wave. Each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave. These wavelets emanating from the wavefront are usually referred to as secondary wavelets and if we draw a common tangent to all these spheres, we obtain the new position of the wavefront at a later time.



 Δ EAC is congruent to Δ BAC; so $\angle i = \angle r$

(ii) Two sources are said to be coherent if the phase difference between them does not change with time.

No, two independent sodium lamps cannot be coherent.

Two independent sodium lamps cannot be coherent as the phase between them does not remain constant with time.

(iii)

$$4\beta_2 = 5\beta_1$$

$$4 \times \frac{\lambda D}{d} = 5 \times \frac{\lambda_{known} D}{d}$$

$$\Rightarrow \lambda = \frac{5}{4} \times \lambda_{known}$$

$$= \frac{5}{4} \times 520$$

$$= 650 \text{ nm}$$





Previous Years' CBSE Board Questions

9.

(i)

11.2 Electron Emission

MCQ

The energy of a photon of wavelength λ is

 (a) hc λ
 (b) hc/λ
 (c) λ/hc
 (d) λh/c
 (2023)

11.3 Photoelectric Effect

VSA (1 mark)

Define the term threshold frequency in photoelectric emission. (2020) (2020)

OR

Define the term "threshold frequency", in the context of photoelectric emission. (Delhi 2019)

 Define the term "Intensity" in photon picture of electromagnetic radiation. (Delhi 2019)

11.4 Experimental Study of Photoelectric Effect

Effect of Intensity of Light on Photocurrent

SAI (2 marks)

8. For the light of wavelength 400 nm incident on the cathode of a photocell, the stopping potential is 6 V. If the wavelength of incident light is increased to 600 nm, calculate the new stopping potential. (Take $h = 4.14 \times 10^{-15}$ eV s) (2021)

ke $h = 4.14 \times 10^{-15}$ eV s) (2021) Monochromatic light of frequency 6.0×10^{14} Hz

is produced by a laser. The power emitted is 2.0×10^{-3} W. Estimate the number of photons emitted per second on an average by the source.

 (ii) Draw a plot showing the variation of photoelectric current versus the intensity of incident radiation on a given photosensitive surface. (Delhi 2014)

SAII (3 marks)

- Light of wavelength 2000 Å is incident on a metal surface of work function 4.2 eV in an experiment on photoelectric effect.
 - (a) Find the maximum kinetic energy (in eV) of ejected photoelectrons.
 - (b) If the intensity of light is doubled, find the change in stopping potential. (2020)
- 11. Plot a graph showing the variation of photoelectric



MCQ

- Photons of energy 3.2 eV are incident on a photosensitive surface. If the stopping potential for the emitted electrons is 1.5 V, the work function for the surface is
 - (a) 1.5 eV (b) 1.7 eV (c) 3.2 eV (d) 4.7 eV (2023)

VSA (1 mark)

- If the frequency of the radiation incident on a photosensitive surface increases (υ > υ₀), how will the stopping potential change? (2021)
- 6. The three curves shown below, represent the variation of photocurrent and applied voltage for two different materials using two different intensities of a monochromatic light. Identify the two curves which are for the same material.



- In photoelectric effect, does the photoelectric current depend on intensity of incident radiation? Give reasons. (2019)
- (a) Define the terms, (i) threshold frequency and (ii) stopping potential in photoelectric effect.
 - (b) Plot a graph of photocurrent versus anode potential for a radiation of frequency υ and intensities l₁ and l₂ (l₁ < l₂). (Delhi 2019)

Effect of Frequency of Incident Radiation on Stopping Potential

VSA (1 mark)

The variation of the stopping potential (V₀) with the frequency
 (υ) of the light incident on two different photosensitive surfaces
 M₁ and M₂ is shown in the figure. Identify the surface which



has greater value of the work function.

current with intensity of light. The work function for the following metals is given.

Na : 2.75 eV and Mo : 4.175 eV.

Which of these will not give photoelectron emission for a radiation of wavelength 3300 Å from a laser beam? What happens if the source of laser beam is brought closer? (Foreign 2016)

- A beam of monochromatic radiation is incident on a photosensitive surface. Answer the following questions giving reasons.
 - (a) Do the emitted photoelectrons have the same kinetic energy?
 - (b) Does the kinetic energy of the emitted electrons depend on the intensity of incident radiation?
 - (c) On what factors does the number of emitted photoelectrons depend? (Foreign 2015)

Effect of Potential on Photoelectric Current

VSA (1 mark)

 Draw graphs showing variation of photoelectric current with applied voltage for two incident radiations of equal frequency and different intensities. Mark the graph for the radiation of higher intensity. (2018)

SAI (2 marks)

- Draw a graph showing variation of photocurrent with the anode potential of a photocell. (1/2, 2020)
- 22. The following graph shows the variation of photocurrent for a photosensitive metal:



- (a) Identify the variable X on the horizontal axis.
- (b) What does the point A on the horizontal axis represent?
- (c) Draw this graph for three different values of frequencies of incident radiation υ₁, υ₂ and υ₃ (υ₁ > υ₂ > υ₃) for same intensity.
- (d) Draw this graph for three different values of intensities of incident radiation l₁, l₂ and l₃

 Estimate the frequency associated with a photon of energy 2 eV. (2019)

SAI (2 marks)

- How would the stopping potential for a given photosensitive surface change if (i) frequency of the incident radiation were increased? and (ii) the intensity of incident radiation were decreased? Justify your answer. (2023)
- 19. Two monochromatic radiations of frequencies υ₁ and υ₂(υ₁ > υ₂) and having the same intensity are in turn, incident on a photosensitive surface to cause photoelectric emission. Explain, giving reason, in which case (i) more number of electrons will be emitted and (ii) maximum kinetic energy of the emitted photoelectrons will be more.

(Delhi 2014C) (Ap)

SAII (3 marks)

- 20. The work function of a metal is 2.31 eV. Photoelectric emission occurs when light of frequency 6.4 × 10¹⁴ Hz is incident on the metal surface. Calculate : (i) the energy of the incident radiation (ii) the maximum kinetic energy of the emitted electron and (iii) the stopping potential of the surface. (2022C)
- 21. Explain giving reasons for the following:
 - (a) Photoelectric current in a photocell increases with the increase in the intensity of the incident radiation.
 - (b) The stopping potential (V₀) varies linearly with the frequency (υ) of the incident radiation for a given photosensitive surface with the slope remaining the same for different surfaces.

(2/3, AI 2017)

(a)
$$\frac{hv}{c^2}$$
 (b) hv (c) $\frac{hc}{E}$ (d) $\frac{hv}{c}$
(2023)

 $(I_1 > I_2 > I_3)$ having same frequency. (AI 2017) [F]

 Describe briefly three experimentally observed features in the phenomenon of photoelectric effect. (2/3, Al 2015)

11.5 Photoelectric Effect and Wave Theory of Light

LA (5 marks)

24. Why is wave theory of electromagnetic radiation not able to explain photoelectric effect? How does photon picture resolve this problem?

(Delhi 2019) (An)

OR

Discuss briefly how wave theory of light cannot explain photoelectric effect. (2/3, Al 2015)

OR

Write three observed features of photoelectric effect which cannot be explained by wave theory of light. (2/5, AI 2015C)

11.6 Einstein's Photoelectric Equation: Energy Quantum of Radiation

MCQ

- 25. A light of frequency v is incident on a metal surface whose work function is W₀. The kinetic energy of emitted electron is K. If the frequency of the incident light is doubled then the kinetic energy of emitted electron will be
 - (a) 2K (b) more than 2K
 - (c) between K and 2K (d) less than K (2023)
- 26. E, c and v represent the energy, velocity and frequency of a photon. Which of the following represents its wavelength?
- 34. Figure shows the stopping potential (V₀) for the photo electron versus (1/λ) graph, for two metals A and B, λ being the wavelength of incident light.



- 27. Photons of energies 1 eV and 2 eV are successively incident on a metallic surface of work function 0.5 eV. The ratio of kinetic energy of most energetic photoelectrons in the two cases will be
 - (a) 1:2 (b) 1:1
 - (c) 1:3 (d) 1:4 (2020)
- If photons of frequency υ are incident on the surfaces of metals A and B of threshold frequencies υ/2 and υ/3 respectively, the ratio of the maximum kinetic energy of electrons emitted from A to that from B is

 (a) 2:3
 (b) 3:4
 - (c) 1:3 (d) √3:√2 (2020) (▲)

VSA (1 mark)

- 29. The threshold wavelength for two photosensitive surfaces A and B are λ₁ and λ₂ respectively. What is the ratio of the work functions of the two surfaces? (2020)
- 30. The figure shows the variation of stopping potential V₀ with the frequency v of the incident radiations for two photosensitive metals P and Q. Which metal has smaller threshold wavelength? Justify your answer.



(AI 2019) Ev

 Name the phenomenon which shows the quantum nature of electromagnetic radiation. (AI 2017) R

SAI (2 marks)

- 1 .
- 32. The figure shows v_m^2 versus $\frac{1}{\lambda}$ graph for photoelectrons emitted from a surface where v_m is the maximum speed of electrons and λ is wavelength of incident radiation. Using this graph and Einstein's photoelectric equation, obtain the expression for Planck's constant and work function of the surface.





- (a) How is the value of Planck's constant determined from the graph?
- (b) If the distance between the light source and the surface of metal A is increased, how will the stopping potential for the electrons emitted from it be effected? Justify your answer.

(2020) Ev

- Write two main observations of photoelectric effect experiment which could only be explained by Einstein's photoelectric equation. (1/2, 2020)
- 36. If light of wavelength 412.5 nm is incident on each of the metals given in table, which one will show photoelectric emission and why?

Metal	Work Function (eV)
Na	1.92
К	2.15
Ca	3.20
Mo	4.17

(2018)

 Using the graph shown in the figure for stopping potential versus the incident frequency of photons, calculate Planck's constant.



SAII (3 marks)

 (a) Give an example each of a metal from which photoelectric emission takes place when irradiated by (i) UV light, (ii) visible light. 33. Why it is the frequency and not the intensity of light source that determines whether emission of photoelectrons will occur or not? Explain.

(Term II 2021-22)

39. (a) Use Einstein's photoelectric equation to depict the variation of the maximum kinetic energy (E_k) of electrons emitted, with the frequency(υ) of the incident radiation.

(b) A photosensitive surface is illuminated with a beam of (i) yellow light, and (ii) red light, both of the same intensity.

In which case will

photoelectrons have more E_k?

(II) more numbers of electrons be emitted?

Justify your answer in each case. (Term II 2021-22)

- The maximum kinetic energy of the photoelectrons emitted is doubled when the wavelength of light incident on the photosensitive surface changes from λ₁ to λ₂. Deduce expressions for the threshold wavelength and work function for the metal surface in terms of λ₁ and λ₂. (2020)
- 41. (i) How does one explain the emission of electrons from a photosensitive surface with the help of Einstein's photoelectric equation?

(ii) The work function of the following metals is given : Na = 2.75 eV, K = 2.3 eV, Mo = 4.17 eV and Ni = 5.15 eV. Which of these metals will not cause photoelectric emission for radiation of wavelength 3300 Å from a laser source placed 1 m away from these metals? What happens if the laser source is brought nearer and placed 50 cm away?

(Delhi 2017) [An]

 (i) State two important features of Einstein's photoelectric equation.

(ii) Radiation of frequency 10^{15} Hz is incident on two photosensitive surfaces *P* and *Q*. There is no photoemission from surface *P*. Photoemission occurs from surface *Q* but photoelectrons have zero kinetic energy. Explain these observations and find the value of work function for surface *Q*.

(Delhi 2017)

- Give reason for maximum kinetic energy of the photoelectrons is independent of the intensity of incident radiation. (1/3, Al 2017)
- Using photon picture of light, show how Einstein's photoelectric equation can be established. Write two features of photoelectric effect which cannot be explained by wave theory. (AI 2017)
- 45. In the study of a photoelectric effect the graph between the stopping potential V and frequency v of the incident radiation on two different metals P and Q is shown in:

(b) The work function of a metal is 4.50 eV. Find the frequency of light to be used to eject electrons from the metal surface with a maximum kinetic energy of 6.06 × 10⁻¹⁹ J. (*Term II 2021-22*)

- (ii) Determine the work function of the metal which has greater value.
- (iii) Find the maximum kinetic energy of electron emitted by light of frequency 8 × 10¹⁴ Hz for this metal.
 (Delhi 2017) (Ap)
- 46. Sketch the graphs showing variation of stopping potential with frequency of incident radiations for two photosensitive materials A and B having threshold frequencies υ_A > υ_B.
 - (i) In which case is the stopping potential more and why?
 - (ii) Does the slope of the graph depend on the nature of the material used ? Explain.

(AI 2016) (An

- 47. Define the term "cut off frequency" in photoelectric emission. The threshold frequency of a metal is f. When the light of frequency 2f is incident on the metal plate, the maximum velocity of photoelectron is v₁. When the frequency of the incident radiation is increased to 5f, the maximum velocity of photoelectrons is v₂. Find the ratio v₁: v₂. (Foreign 2016)
- Write three characteristic features in photoelectric effect which cannot be explained on the basis of wave theory of light, but can be explained only using Einstein's equation. (Delhi 2016)

OR

Explain how Einstein's photoelectric equation is used to describe photoelectric effect satisfactorily.

(3/5, AI 2015C) (U)

49. Write Einstein's photoelectric equation and mention which important features in photoelectric effect can be explained with the help of this equation.

The maximum kinetic energy of the photoelectrons gets doubled when the wavelength of light incident on the surface changes from λ_1 to λ_2 . Derive the expressions for the threshold wavelength λ_0 and work function for the metal surface. (Delhi 2015)

LA (5 marks)

- 50. Figure shows a plot of stopping potential (V₀) with frequency (υ) of incident radiation for two photosensitive material M₁ and M₂. Explain
 - (i) why the slope of both the lines is same?
 - (ii) for which material emitted electrons have greater kinetic energy for the same frequency







(i) Which one of the two metals has higher threshold frequency?

11.7 Particle Nature of Light : The Photon

VSA (1 mark)

- What is the wavelength of a photon of energy 3.3 × 10⁻¹⁹ J? (2020)
- Define intensity of radiation on the basis of photon picture of light. Write its SI unit. (AI 2014)

SAI (2 marks)

 Write three basic properties of photons which are used to obtain Einstein's photoelectric equation. Use this equation to draw a plot of maximum kinetic energy of the electrons emitted versus the frequency of incident radiation. (Al 2014C)

SAII (3 marks)

- 54. In the wave picture of light, intensity of light is determined by the square of the amplitude of the wave. What determines the intensity in the photon picture of light? (2/3, AI 2016)
- (a) Write the important properties of photons which are used to establish Einstein's photoelectric equation.

(b) Use this equation to explain the concept of (i) threshold frequency and (ii) stopping potential.

(AI 2015)

11.8 Wave Nature of Matter

MCQ

56. A photon and an alpha particle have the same kinetic energy. The ratio of de Broglie wavelengths associated with the proton to that with the alpha particle is

(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) $\frac{1}{2}$ (2023) of incident radiation?



(3/5, AI 2015C) EV

- How will the de Broglie wavelength associated with an electron be affected when the (i) velocity of the electron decreases? and (ii) accelerating potential is increased? Justify your answer. (2023)
- 59. A heavy particle initially at rest splits spontaneously into two particles of masses m₁ and m₂ having nonzero velocities. The ratio of de Broglie wavelengths associated with the particles is
 - (a) m_1/m_2 (b) m_2/m_1
 - (c) 1 (d) $\sqrt{m_2} / \sqrt{m_1}$ (2020)

VSA (1 mark)

- An electron is accelerated through a potential difference of 100 V. Calculate the de Broglie wavelength associated with it. (2021)
- A proton and a deuteron are moving with the same speed. Find the ratio of the de Broglie wavelength (λ_p/λ_d) associated with them. (2021)
- A proton and an electron have equal speeds. Find the ratio of de Broglie wavelengths associated with them. (2020)
- 63. The graph showing the correct variation of linear momentum (p) of a charge particle with its de-Broglie wavelength (λ) is



57. Which of the following graphs correctly represents the variation of a particle momentum with its associated de-Broglie wavelength?



- An α-particle and a proton are accelerated through the same potential difference. Find the ratio of their de Broglie wavelengths. (Delhi 2017)
- 68. The wavelength λ of a photon and the de Broglie wavelength of an electron have the same value. Show that energy of a photon is (2λ mc/h) times the kinetic energy of electron, where m, c and h have their usual meaning. (Foreign 2016) (An)
- A proton and an α-particle have the same de-Broglie wavelength. Determine the ratio of
 - (i) their accelerating potentials
 - (ii) their speeds. (Delhi 2015)
- A proton and a deuteron are accelerated through the same accelerating potential. Which one of the two has
 - (a) greater value of de-Broglie wavelength associated with it, and
 - (b) less momentum?

Give reasons to justify your answer. (Delhi 2014)

- 71. X-rays fall on a photosensitive surface to cause photoelectric emission. Assuming that the work function of the surface can be neglected, find the relation between the de-Broglie wavelength (λ) of the electrons emitted and the energy (E_{ν}) of the incident photons. Draw the nature of the graph for λ as a function of E_{ν} . (Delhi 2014C)
 - SAII (3 marks)
- 72. (a) A proton and α-particle are moving with the same speed. Which one of them has greater value

 Draw a plot showing the variation of de-Broglie wavelength of electron as a function of its K.E.

(Delhi 2015C) 🕕

SAI (2 marks)

- 65. What are matter waves? A proton and an alpha particle are accelerated through the same potential difference. Find the ratio of the de Broglie wavelength associated with the proton to that with the alpha particle. (Term II 2021-22)
- (a) Calculate the energy and momentum of a photon in a monochromatic beam of wavelength 331.5 nm.

(b) How fast should a hydrogen atom travel in order to have the same momentum as that of the photon in part (a) ? (Term II 2021-22)

of de Broglie wavelength associated with it? Justify your answer.

(b) Lights of wavelengths 430 nm, 450 nm and 660 nm are incident on a metal surface whose threshold wavelength is 600 nm. In which case/cases will photoemission take place and why? Calculate the threshold frequency of the metal surface. (2022C)

- Obtain an expression for the ratio of the accelerating potentials required to accelerate a proton and an α-particle to have the same de-Broglie wavelength associated with them. (2/3, AI 2019)
- (a) An electron and a proton are accelerated through the same potential. Which one of the two has
 - greater value of de-Broglie wavelength associated with it, and
 - (ii) lesser momentum? Justify your answer in each case.
 - (b) How is the momentum of a particle related with its de-Broglie wavelength? Show the variation on a graph. (AI 2019)
- 75. An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de Broglie wavelength associated with the electrons. Taking other factors, such as numerical aperture etc. to be same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light? (AI 2014)





CBSE Sample Questions

11.2 Electron Emission

MCQ

- Photoelectric emission from a given surface of metal can take place when the value of a 'physical quantity' is less than the energy of incident photon. The physical quantity is
 - (a) threshold frequency
 - (b) work function of surface
 - (c) threshold wavelength
 - (d) stopping potential

(2019-20) (1)

11.3 Photoelectric Effect

MCQ

- The work function for a metal surface is 4.14 eV. The threshold wavelength for this metal surface is
 - (a) 4125 Å (b) 2062.5 Å
 - (c) 3000 Å (d) 6000 Å (2022-23)

11.5 Photoelectric Effect and Wave Theory of Light

SAII (3 marks)

 State the main implications of observations obtained from various photoelectric experiments. Can these implications be explained by wave nature of light? Justify your answer. (2020-21) (Ap)

11.6 Einstein's Photoelectric Equation: Energy Quantum of Radiation

MCQ

- Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.
 - (a) Both A and R are true and R is the correct explanation of A.
 - (b) Both A and R are true and R is NOT the correct explanation of A.

11.4 Experimental Study of Photoelectric Effect

SAII (3 marks)

The graph shows the variation of photocurrent for a photosensitive metal.



- (a) What does X and A on the horizontal axis represent?
- (b) Draw this graph for three different values of frequencies of incident radiation υ₁, υ₂ and υ₃ (υ₃ > υ₂ > υ₁) for the same intensity.
- (c) Draw this graph for three different values of intensities of incident radiation l₁, l₂ and l₃ (l₃> l₂>l₁) having the same frequency. (2022-23)

SAII (3 marks)

- Light of wavelength 2000 Å falls on a metal surface of work function 4.2 eV.
 - (a) What is the kinetic energy (in eV) of the fastest electrons emitted from the surface?
 - (b) What will be the change in the energy of the emitted electrons if the intensity of light with same wavelength is doubled?
 - (c) If the same light falls on another surface of work function 6.5 eV, what will be the energy of emitted electrons? (Term II 2021-22)

11.7 Particle Nature of Light : The Photon

SAII (3 marks)

 Radiation of frequency 10¹⁵ Hz is incident on three photosensitive surfaces A, B and C. Following observations are recorded:

Surface A : no photoemission occurs

Surface B : photoemission occurs but the photo-electrons have zero kinetic energy.





- (c) A is true but R is false.
- (d) A is false and R is also false.

Assertion (A) : The photoelectrons produced by a monochromatic light beam incident on a metal surface have a spread in their kinetic energies.

Reason (R) : The energy of electrons emitted from the metal surface, is lost in collision with the other atoms in the metal. (2022-23)

VSA (1 mark)

 In a photoelectric experiment, the potential required to stop the ejection of electrons from cathode is 4 V. What is the value of maximum kinetic energy of emitted Photoelectrons? (2020-21) Surface C: photo emission occurs and photoelectrons have some kinetic energy.

Using Einstein's photo-electric equation, explain the three observations. (2022-23)

11.8 Wave Nature of Matter

SAII (3 marks)

- (a) Explain de-Broglie argument to propose his hypothesis. Show that de-Broglie wavelength of photon equals electromagnetic radiation.
- (b) If, deuterons and alpha particle are accelerated through same potential, find the ratio of the associated de-Broglie wavelengths of two.

(2020-21) An

Detailed SOLUTIONS

9.

Previous Years' CBSE Board Questions

- (b): Energy of photon of wavelength λ, E = hu
- \therefore $c=\upsilon\lambda$ or $\upsilon=\frac{c}{\lambda}$ or $E=\frac{hc}{\lambda}$

 Threshold frequency is defined as the minimum frequency of incident radiation below which the photoelectric emission does not occur.

 The amount of light energy or photon energy incident per unit area per unit time is called intensity of electromagnetic radiation.

The number of photoelectrons emitted, i.e., photoelectric current depends only upon its intensity.

The photo current is directly proportional to the number of photoelectrons emitted per second. This implies that the number of photoelectrons emitted per second is directly proportional to the intensity of incident radiation.



8.
$$\lambda_1 = 400 \text{ nm} = 4 \times 10^{-7} \text{ m}$$

 $\lambda_2 = 600 \text{ nm} = 6 \times 10^{-7} \text{ m}$
 $h = 4.14 \times 10^{-15} \text{ eV s}$
 $V_{01} = 6 \text{ V}$
 $V_{02} = ?$
 $eV_0 = hv - W_0$
 $eV_0 = hv_1 - W$
 $eV_0 = hv_2 - W$
 $\therefore e(V_{01} - V_{02}) = h(v_1 - v_2) = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$
 $(6 - V_{02}) = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{16 \times 10^{-19} \times 6.624 \times 10^{18}} \left(\frac{1}{4 \times 10^{-7}} - \frac{1}{6 \times 10^{-7}}\right)$

4. (b): According to Einstein's photoelectric equation,

 $eV = hv - \phi$

 $\phi = hv - eV$

Here, energy of photon = 3.2 eV Stopping potential = 1.5 eV

∴ φ = 3.2 eV - 1.5 eV = 1.7 eV

 If the frequency of incident radiation increase then the kinetic energy of ejected photoelectron will also increase.

Hence, we have need to increase the stopping potential.

Curve 1 and 2 are for same materials. The value of stopping potential increases with the frequency of incident radiation.

> Photoelectric current

> > Intensity of light ----

11. Variation of photoelectric current with intensity of light for a given frequency of incident radiation

Given that $\lambda = 3300 \times 10^{-10}$ m, $\phi_{Na} = 2.75$ eV, $\phi_{Mo} = 4.175$ eV Then energy of the laser beam is

$$F = \frac{hc}{c} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{c}$$

$$=\frac{n}{\lambda}=\frac{0.0\times10^{-10}\times1.6\times10^{-19}}{3300\times10^{-10}\times1.6\times10^{-19}}=3.75\,\text{eV}$$

Since $E < \phi_{Mo}$ therefore there will be no emission of photoelectrons for molybdenum (Mo).

Bringing the source nearer will cause to emit more photoelectrons as intensity on the plate will increase.

Concept Applied

Work function is the minimum energy or work done to free the electrons or remove the electrons from the surface of a metal.

12. (a) Yes, all emitted photoelectrons have same kinetic energy as the kinetic energy of emitted photoelectrons depends upon frequency of the incident radiation for a given photosensitive surface.

(b) No, the kinetic energy of emitted electrons does





$$6 - V_{02} = 0.975$$

 $V_{02} = 6 - 0.975 = 5.025 V$

Given, υ = 6.0 × 10¹⁴ Hz

Answer Tips 💋

The ratio of the power emitted by the source and the energy of the photon gives the number of photons emitted per second by the source.

 $P = 2.0 \times 10^{-3} W$

Let n is the number of photons emitted by the source per second.

 $n = \frac{P}{E} = \frac{P}{hv} = \frac{2 \times 10^{-3}}{6.63 \times 10^{-34} \times 6.0 \times 10^{14}}$ = 0.0502 × 10¹⁷ = 5 × 10¹⁵ photons per second. (ii) $\frac{10}{1000} = \frac{1}{1000} = \frac{1}{1000} = \frac{hc}{\lambda} = W_0$ 10. (a) K.E. = $hv - W_0 = \frac{hc}{\lambda} = W_0$ $\lambda = 2000 \text{ Å} = 2 \times 10^{-7} \text{ m}, W_0 = 4.2 \text{ eV}$ K.E. = $\left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7} \times 1.6 \times 10^{-19}}\right) - 4.2$ = 6.2 - 4.2 eV = 2 eV

(b) Stopping potential,
$$eV_0 = \frac{1}{2}mv^2 = K.E.$$

Stopping potential does not depend upon the intensity of light. Hence, if intensity is doubled, the stopping potential remains constant.

16. M₂ has greater value of work function, since it has higher value of threshold frequency.

Concept Applied

Stopping potential is independent of the intensity of incidence radiation of a given frequency.

17. Energy of photon, E = hv $E = 2 \text{ eV} = 2 \times 1.6 \times 10^{-19}$ $v = \frac{E}{h} = \frac{2 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 4.8 \times 10^{14} \text{ Hz}$

18. (b) $hv = hv_0 + eV_0$

 As, incident frequency υ increases, the stopping potential also increases.

(ii) There is no effect on stopping potential as the intensity changes. The photoelectric current will change as the intensity change.

 Intensity = Number of photons per unit area per unit time not depend on the intensity of incident radiation. If the intensity is increased, number of photons will also increase but energy of each photon remains same as the frequency is also same. The maximum kinetic energy depends on frequency not on intensity.

(c) The number of emitted photoelectrons depends only on intensity of incident light. For a given frequency of incident radiation, its intensity depends on the number of photons.



shown in graph.

Photoelectric current ~ Intensity Photoelectric current



 $-V_0$ Anode potential **15.** (a) (i) Threshold Frequency: The minimum frequency of incident light which is just capable of ejecting electrons from a metal is called the threshold frequency. It is denoted by v_0 .

(ii) Stopping Potential : The minimum retarding potential applied to anode of a photoelectric tube which is just capable of stopping photoelectric current is called the stopping potential. It is denoted by V_0 (or V_S).



 (a) The number of photoelectrons emitted, *i.e.*, photoelectric current depends only upon its intensity.

The photo current is directly proportional to the number of photoelectrons emitted per second. This implies that the number of photoelectrons emitted per second is directly proportional to the intensity of incident radiation.



(b) The energy of the emitted electrons depends on the frequency of the incident radiations. The stopping potential is more negative for higher frequencies of incident radiation. From the figure, stopping potentials are in order $V_{03} > V_{02} > V_{01}$ if the frequencies are in the order of $v_3 > v_2 > v_1$. This implies that greater the frequency of incident light, greater is the maximum kinetic energy of the photoelectrons.





For unit area and unit time, $l_1 = l_2 \Rightarrow n_1 \upsilon_1 = n_2 \upsilon_2$ $\frac{n_2}{n_1} = \frac{\upsilon_1}{\upsilon_2} > 1 \Rightarrow n_2 > n_1$

For same intensity number of photons per unit area per unit time is large for v_2 *i.e.* n_2 . Hence, more electrons will be emitted corresponds to v_2 .

(ii) The maximum kinetic energy of emitted electrons is more for the light of greater frequency. Since $v_1 > v_2$, maximum kinetic energy of emitted photoelectrons will be correspond to v_1 .

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23. On the basis of experiments on photoelectric effect, three observed features are :

(i) Emission of photoelectrons start as soon as light falls on metal surface *i.e.*, there is no time lag between incidence of light and emission of photoelectrons.

(ii) The emission of photoelectrons takes place only when the frequency of the incident radiations is above a certain critical value called threshold frequency v_0 , which is characteristic of that metal emitting electrons.

Above threshold frequency vo, maximum kinetic energy



 (a) The variable X on the horizontal axis is collector plate potential.

(b) The point A on the horizontal axis represents stopping potential.



 In the interaction of photons with free electrons, the entire energy of photon is absorbed.

(ii) Energy of photon is directly proportional to frequency. Intensity of incident radiation depends on the number of photons falling per unit area per unit time for a given frequency.

Key Points 🚫

 Free electrons in a metal do not have the same energy. At constant temperature, each electron has its own





with which photoelectrons are emitted is directly proportional to frequency v of incident radiation. So, the graph plotted between $(1/2)mv_{max}^2$ or eV_0 with frequency v is a straight line for frequencies above threshold frequency v_0 .

(iii) The maximum kinetic energy with which a photoelectron is emitted from a metallic surface is independent of the intensity of light and depends only upon its frequency.

 The observed characteristics of photoelectric effect could not be explained on the basis of wave theory of light.

(i) According to wave theory, the light propagates in the form of wavefronts and the energy is distributed uniformly over the wavefronts. With increase of intensity of light, the amplitude of waves and the energy stored by waves will increase. These waves will then, provide more energy to electrons of metal, consequently the energy of electrons will increase.

Thus, according to wave theory, the kinetic energy of photoelectrons must depend on the intensity of incident light, but according to experimental observations, the kinetic energy of photoelectrons does not depend on the intensity of incident light.

 According to wave theory, the light of any frequency can emit electrons from metallic surface provided the intensity of light be sufficient to provide necessary energy for emission of electrons, but according to experimental

observations, the light of frequency less than threshold frequency can not emit electrons, whatever be the intensity of incident light.

(iii) According to wave theory, the energy transferred by light waves will not go to a particular electrons, but it will be distributed uniformly to all electrons present in the illuminated surface. Therefore, electrons will take some time to collect the necessary energy for their emission. The time for emission will be more for light of less intensity and vice versa. But experimental observations show that the emission of electrons take place instantaneously after the light is incident on the metal; whatever be the intensity of light.

So, we conclude that wave nature of light cannot be used to explain photoelectric effect.

The photon picture resolve this problem by following features. certain energy. Because of this energy required to eject an electron is different for different electrons residing on metal surface.

(b): Kinetic energy of emitted electron

 $K = hv - W_0$...(i) $hv = K + W_0 \text{ or } = hv - W_0$...(ii)

Work function is constant for every metal surface and if we double the frequency of incident light, then new KE will be

 $K' = h(2v) - W_0 = 2hv - W_0$

Using equation (ii), we get

 $K' = 2(K + W_0) - W_0 = 2K + W_0$

 Kinetic energy will be more than 2K, hence option (b) is correct.

$$26. (c): E = hv = \frac{hc}{\lambda}$$

$$\lambda = \frac{nc}{r}$$

27. (c):
$$(KE)_{max} = hv - \phi$$

$$\frac{(KE)_1}{2} \frac{hv_1 - \phi}{2} \frac{1 - 0.5}{2} \frac{0.5}{2} = 0.5$$

28. (b) :
$$E = hv = hv_{0A} + (KE)_{max_A} = hv_{0B} + (KE)_{max_B}$$

1

$$\therefore \quad h\upsilon = \frac{h\upsilon}{2} + K_A = \frac{h\upsilon}{3} + K_B \Rightarrow \frac{K_A}{K_B} = \frac{h\upsilon}{2} \times \frac{3}{2h\upsilon} = \frac{3}{4}$$

29. Work function
$$=hv = \frac{hc}{\lambda}$$

$$\therefore \text{ The ratio, } \frac{\phi_A}{\phi_B} = \frac{hc}{\lambda_A} \times \frac{\lambda_B}{hc} = \frac{\lambda_2}{\lambda_1}$$

30. Metal Q has smaller threshold wavelength.

 $eV_0 = hv - hv_0$ or, $V_0 = \left(\frac{h}{e}\right)v - \frac{h}{e}v_0$

For P, when $V_0 = 0$, $v_p = v_0 = 0.1 \times 10^{15} \text{ s}^{-1}$ For Q, when $V_0 = v$, $v_Q = v_0 = 1 \times 10^{15} \text{ s}^{-1}$ As $v_Q > v_p \text{ so } \lambda_Q < \lambda_p$.

 Photoelectric effect shows the quantum nature of electromagnetic radiation.

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32. Consider the graph shown below Einstein's photoelectric equation v_m^2 $KE = hv - \phi_0$...(i) $\mathsf{KE} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \quad \left[\because \upsilon = \frac{c}{\lambda} \right]$ $(1/\lambda_0)$ $\left(\frac{1}{\lambda}\right)$ $\frac{1}{2}mv_m^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$ $v_m^2 = \frac{2hc}{m\lambda} - \frac{2hc}{m\lambda_0}$ $v_m^2 = \left(\frac{2hc}{m}\right) \frac{1}{\lambda} - \left(\frac{2hc}{m}\right) \frac{1}{\lambda_0}$...(ii) (i) Now from graph, Slope, $\frac{2hc}{m} = \tan\theta$ $\frac{2hc}{m} = \tan\theta$ $h = \frac{m \tan \theta}{2c}$...(iii) (ii) Now, for work function

$$\phi = \frac{nc}{\lambda_0}$$

Therefore, from graph, (1/h_o) is threshold wavelength on

$$\frac{1}{\lambda} \operatorname{graph}_{\phi_0} = \frac{hc}{\lambda_0}$$

Now take value from eq.(iii)

 $\phi_0 = \frac{m}{2c} \tan\theta \times \frac{c}{\lambda_0} \Rightarrow \frac{m \tan\theta}{2\lambda_0}$

33. Below threshold frequency vo, energy of photon is less than work function of metal surface, i.e. less than minimum amount of energy required to liberate an electron. So emission of photoelectrons take place only when the frequency of incident radiation is above or equal to the threshold frequency up.





Slope of the line $=\frac{h}{c}$ *i.e.*, it is a constant quantity and 2.

does not depend on nature of metal surface.

P

Intercept of graph 1 on the stopping potential axis (ii)

$$=\frac{\text{work function}(W)}{=}-\frac{hv_0}{V}$$

e

2 Intercept of the line depends upon the stopping function of the metal surface.

(b) Stopping potential only depends on the frequency of incident light. If distance increases, it does not affect the stopping potential.

Concept Applied

⇒ V_0 versus v curve is a straight line with slope $\left(=\frac{h}{e}\right)$, independent of the nature of the material.

 On the basis of experiments on photoelectric effect, two observed features are :

The emission of photoelectrons takes place only (i) when the frequency of the incident radiations is above a certain critical value called threshold frequency uo, which is characteristic of that metal emitting electrons.

Above threshold frequency v₀, maximum kinetic energy with which photoelectrons are emitted is directly proportional to frequency u of incident radiation.

(ii) The maximum kinetic energy with which a photoelectron is emitted from a metallic surface is independent of the intensity of light and depends only upon its frequency.

36. Wavelength of incident light, λ = 412.5 nm Energy of incident light, $E = \frac{hc}{h} = \frac{1242 \text{ eV nm}}{412 \text{ From}} = 3 \text{ eV}$

Metals Na and K will show photoelectric emission because their work functions are less than the energy of incident light.





The slope of graph is, $\tan \theta = \frac{V_0}{(v - v_0)} = \frac{eV_0}{e(v - v_0)}$

As energy of a photon depends on the frequency of light, so the maximum kinetic energy with which photoelectron is emitted depends only on the energy of photon or on the frequency of incident radiation.

34. (a) The graph showing the variation of stopping potential (V_0) with the frequency of incident radiation (v)for two different photosensitive materials having work functions W_1 and W_2 ($W_1 > W_2$) is shown in figure.

 (a) (i) Zinc, cadmium (ii) Lithium, sodium (b) Work function of metal, φ = 4.50 eV Kinetic energy = 6.06 × 10⁻¹⁹ J $=\frac{6.06\times10^{-19}}{1.6\times10^{-19}}$ eV = 3.78 eV Now, K.E. = hu - 6 ⇒ hu = K.E. + φ = 3.78 eV + 4.50 eV = 8.28 eV $\frac{8.28 \text{ eV}}{h} = \frac{8.28 \text{ eV}}{4.135 \times 10^{-15} \text{ eV s}} = 2.002 \times 10^{15} \text{Hz}$

 (a) According to Einstein photoelectric equation, $E_k = h \upsilon - h \upsilon_0$

Where, up is threshold frequency and h is Planck's constant.



(b) Yellow light has high frequency (thus more energy) than red light.

Yellow light will emit photoelectrons having more E_k. (I) (II) Number of photoelectrons emitted per second is directly proportional to intensity of incident light. Since yellow light and red light have same intensity number of electrons emitted will be same.

40. Since
$$\frac{hc}{\lambda} = K_{\max} + \phi$$

For case I: $\frac{hc}{\lambda_1} = K + \frac{hc}{\lambda_T} \Rightarrow hc \left[\frac{1}{\lambda_1} - \frac{1}{\lambda_T}\right] = K$...(i)

For case II:
$$\frac{hc}{\lambda_2} = 2K + \frac{hc}{\lambda_T} \Rightarrow hc \left[\frac{1}{\lambda_2} - \frac{1}{\lambda_T}\right] = 2K$$

Substituting (i) in (ii),

$$hc\left[\frac{1}{\lambda_{2}} - \frac{1}{\lambda_{T}}\right] = 2hc\left[\frac{1}{\lambda_{1}} - \frac{1}{\lambda_{T}}\right] \Rightarrow \lambda_{T} = \frac{\lambda_{1}\lambda_{2}}{2\lambda_{2} - \lambda_{1}}$$

Work function, $\phi = \frac{hc}{\lambda_{T}} = \frac{hc(2\lambda_{2} - \lambda_{1})}{\lambda_{1}\lambda_{2}}$

 (i) The Einstein's photoelectric equation is given as $K_{max} = hv - \phi_0$

A12

Concept Applied (@)

The condition for the occurrence of photoelectric effect is that the energy associated with the photon should be greater than the work function of the metal.

Using Einstein's photoelectric equation,

 $eV = hv - \phi$

On differentiation, we get $e\Delta V = h\Delta v$

or
$$h = \frac{e\Delta V}{\Delta v} = \frac{1.6 \times 10^{-19} \times (1.23 - 0)}{(8 - 5) \times 10^{14}} = 6.56 \times 10^{-34} \text{ Js}$$

for λ = 3300 Å

 $\frac{hc}{\lambda} = \frac{1.989 \times 10^{-25}}{3300 \times 10^{-10}} = \frac{6.03 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.77 \text{ eV}$

Mo and Ni will not cause photoelectric emission. 24

If the laser source is brought nearer and placed 50 cm away, then photoelectric emission will not effect, since it depends upon the work function and threshold frequency.

 Two features of Einstein's photoelectric equation:

 Below threshold frequency υ_o corresponding to W₀. no emission of photoelectrons takes place.

(b) As the number of photons in light depend on its intensity, and one photon liberates one photoelectron. So number of emitted photoelectrons depend only on the intensity of incident light for a given frequency.

Below threshold frequency no emission takes place. As there is no photoemission from surface P i.e., the frequency of incident radiation is less than the threshold

frequency for surface P.

From surface Q photoemission is possible i.e., the frequency of incident radiation is equal or greater than threshold frequency. As the kinetic energy of photo electrons is zero i.e., the energy of incident radiation is just sufficient to pull out the electron from the surface Q.

Work function for surface Q, $W_Q = hv$. As K.E. = 0 ; $v = v_0 = 10^{15}$ Hz $W_Q = 6.6 \times 10^{-34} \times 10^{15} = 6.6 \times 10^{-19}$ J = 4.125 eV

 For a given frequency of the incident radiation, the stopping potential is independent of its intensity, i.e., the maximum kinetic energy of photoelectrons depends on the light source and emitter plate material but is independent of intensity of radiation. As Kmax = eVo where V₀ = Stopping potential

44. According to photon picture of light, in photoelectric effect, electron absorbs a quantum of energy (hu) of radiation. If this quantum of energy absorbed exceeds the minimum energy needed for the electron to escape from the metal surface, the electron is emitted with some kinetic energy, the maximum value of which can be given

$$K_{max} = h\upsilon - \phi_0$$



by

...(iii)



Since K_{max} must be non-negative implies that photoelectric emission is possible only if $hv > \phi_0$

or
$$v_f > v_0$$
 where $v_0 = \frac{\phi_0}{h}$.

This shows that the greater the work function ϕ_0 , higher the threshold frequency υ_0 needed to emit photoelectrons.

Thus, there exists a threshold frequency $v_0 = \frac{\phi_0}{h}$ for the

metal surface, below which no photoelectric emission is possible.

(ii) Condition for photoelectric emission, $h\upsilon > \phi_0$

or
$$\frac{hc}{\lambda} > \phi_0$$

(iii) The maximum kinetic energy, of electron emitted by light of frequency 8 × 10¹⁴ Hz is,

$$\therefore K_{\text{max}} = h(\upsilon - \upsilon_0) = 6.6 \times 10^{-34} (8 \times 10^{14} - 6 \times 10^{14}) = 13.2 \times 10^{-20} \text{ J} = 0.825 \text{ eV}$$

We know,

 $K_{\max} = eV_s = h(\upsilon - \upsilon_0)$

or,
$$V_s = \frac{h}{e}\upsilon - \frac{h}{e}\upsilon_0$$

(i) From the graph for the same value of υ, stopping potential is more for material B.

as
$$V_s = \frac{h}{e}(v - v_0)$$

 $\therefore V_s$ is higher for lower value
of v_0 . Here $v_B < v_A$
so $V_{sB} > V_{sA}$.

(ii) Slope of the graph is given by $\frac{h}{e}$ which is constant

for all the materials. Hence slope of the graph does not depend on the nature of the material used.

47. The minimum value of the frequency of light below which the photoelectric emission stops completely, howsoever large may be the intensity of light, is called the cut-off frequency.

Given that threshold frequency of metal is f and frequency of light is 2f. Using Einstein's equation for photoelectric effect, we can write

$$h(2f - f) = \frac{1}{2} m v_1^2 \qquad ...(i)$$

Similarly, for light having frequency 5f, we have

$$h(5f-f) = \frac{1}{2} mv_2^2$$
 ...(ii)

Using (i) and (ii), we find

$$\frac{f}{4f} = \frac{v_1^2}{v_2^2} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{1}{4}} \Rightarrow \frac{v_1}{v_2} = \frac{1}{2} .$$

This is known as Einstein's photoelectric equation. ϕ_0 is the work function of metal, which is the minimum energy needed by a surface electron to come out.

The two features of photoelectric effect which cannot be explained by wave theory, are

(i) The instantaneous emission of electrons

(ii) The existence of threshold frequency for a metal surface

45. (i) For P, threshold frequency $v_P = 3 \times 10^{14}$ Hz For Q, threshold frequency $v_Q = 6 \times 10^{14}$ Hz So, metal Q has higher threshold frequency.

(ii) Work function for Q,

$$W_Q = hv_Q = 6.6 \times 10^{-34} \times 6 \times 10^{14} = 39.6 \times 10^{-20} \text{ J}$$

 $= 2.47 \text{ eV}$

so number of photoelectrons emitted depend only on its intensity.

49. Einstein's photoelectric equation

$$K_{\text{max}} = \frac{1}{2} mv^2 = hv - \phi_0 = hv - hv_0$$
 ...(i)

W₀ = work function of the target metal

Three salient features observed are

(i) Below threshold frequency v_0 corresponding to W_0 , no emission of photoelectrons takes place.

(ii) As energy of a photon depends on the frequency of light, so the maximum kinetic energy with which photoelectron is emitted depends only on the energy of photon or on the frequency of incident radiation.

(iii) For a given frequency of incident radiation, intensity of light depends on the number of photons per unit area per unit time and one photon liberates one photoelectron, so number of photoelectrons emitted depend only on its intensity.

$$K_{\text{max}} = \frac{hc}{\lambda} - \phi_0$$

According to question,

$$K_{\max} = \frac{hc}{\lambda_1} - \phi_0 \qquad \dots (ii)$$

$$2K_{\max} = \frac{hc}{\lambda_2} - \phi_0 \qquad ...(iii)$$

From eqn. (ii) and (iii),

$$2\left(\frac{hc}{\lambda_{1}}-\phi_{0}\right) = \frac{hc}{\lambda_{2}}-\phi_{0}$$

$$\phi_{0} = \frac{2hc}{\lambda_{1}}-\frac{hc}{\lambda_{2}} = hc\left(\frac{2}{\lambda_{1}}-\frac{1}{\lambda_{2}}\right)$$
Also, $\phi_{0} = \frac{hc}{\lambda_{0}} \therefore \frac{hc}{\lambda_{0}} = hc\left(\frac{2}{\lambda_{1}}-\frac{1}{\lambda_{2}}\right)$

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48. Einstein's photoelectric equation is given below.

 $hv = \frac{1}{2}mv_{max}^2 + W_0$

where v = frequency of incident radiation

 $\frac{1}{2}mv_{max}^2$ = maximum kinetic energy of an emitted electron

W₀ = work function of the target metal

Three salient features observed are

(i) Below threshold frequency v_0 corresponding to work function W_0 , no emission of photoelectrons takes place.

(ii) As energy of a photon depends on the frequency of light, so the maximum kinetic energy with which photoelectron is emitted depends only on the energy of photon or on the frequency of incident radiation.

(iii) For a given frequency of incident radiation, intensity of light depends on the number of photons per unit area per unit time and one photon liberates one photoelectron,



 The amount of light energy or photon energy, incident per unit area per unit time is called intensity of radiation.

SI Unit : W m⁻² or J m⁻² s⁻¹.

53. Photons : According to Planck's quantum theory of radiation, an electromagnetic wave travels in the form of discrete packets of energy called quanta.

The main features of photons are as follows:

 In the interaction of photons with free electrons, the entire energy of photon is absorbed.

(ii) Energy of photon is directly proportional to frequency. Intensity of incident radiation depends on the number of photons falling per unit area per unit time for a given ≤ frequency.

υ,

φ,

υ-

Frequency of incident

radiation

(iii) In photon electron collision, the total energy and momentum remain constant. Einstein's photoelectric equation is

 $K_{\rm max} = h\upsilon - \phi_0$

Work function,
$$W_0 = \frac{hc(2\lambda_2 - \lambda_1)}{\lambda_1 \cdot \lambda_2}$$

50. (i) Slope of line = $\frac{\Delta V}{\Delta \upsilon}$ [:: $e\Delta V = h\Delta \upsilon$]

Slope of line = $\frac{h}{a}$

⇒ It is a constant quantity and does not depend on nature of metal surface.

(ii) Maximum kinetic energy of emitted photoelectron,

$$KE = eV_0 = hv - hv_0$$

...(i)

For a given frequency $V_1 > V_2$ (from the graph) So, from equation (i), $(KE)_1 > (KE)_2$

Since the metal M_1 has smaller threshold frequency *i.e.*, smaller work function. It emits electrons having a larger kinetic energy.

electrons of atoms of metal on its surface. Energy $h_{\rm D}$ of each photon is partially utilized by an electron to become free or to overcome its "work function" W_0 and rest of the absorbed energy provides the maximum kinetic energy to the photoelectron during the emission. *i.e.*

$$hv = \frac{1}{2}mv_{\max}^2 + W_0$$

The minimum value of the frequency of incident radiation below which the photoelectric emission stops *i.e.* kinetic energy of photoelectron is zero is called threshold frequency (v_0).

Threshold frequency,
$$v_0 = \frac{W}{h}$$

 $\frac{1}{2}mv_{max}^2 = K.E._{max} = hv - W_0$
or, K.E._max = eV_0

When work done by collecting electrode potential on a photoelectron is equal to its maximum kinetic energy then the electrode potential is known as stopping potential.

Stopping potential,
$$V_0 = \frac{K.E_{max}}{e}$$

56. (b): De Broglie wavelength is given as

$$\lambda = \frac{h}{p}$$

But, $p = \sqrt{2m}$







 For a given frequency, intensity of light in the photon picture is determined by

 $l = \frac{\text{Energy of photons}}{\text{area} \times \text{time}} = \frac{n \times h_0}{A \times t}$

where n is the number of photons incident normally on crossing area A in time t.

Commonly Made Mistake (A

Sometime students define intensity as energy per unit time whereas actually it is energy per unit area per unit time. To avoid such confusion, go through its definition.

55. (a) The main features of photons are as follows:(i) In the interaction of photons with free electrons, the entire energy of photon is absorbed.

 (ii) Energy of photon is directly proportional to frequency.
 Intensity of incident radiation depends on the number of photons falling per unit area per unit time for a given frequency.
 (iii) In photon electron collision, the total energy and momentum remain constant.



Einstein's photoelectric equation is

 $K_{\rm max} = h\upsilon - \phi_0$

(b) Einstein's photoelectric equation : According to Einstein, when light is incident on metal surface, incident photons are absorbed completely by valence

60. An electron accelerated through a potential difference = 100 V

De-broglie wavelength associated with electron $\lambda = \frac{12.27}{\sqrt{2}}$ Å

$$=\frac{12.27}{\sqrt{100}}$$
Å = 1.227 Å

A proton and a deutron moving with same speed = v₀
 Let mass of proton = m_p

Wavelength of proton associated with it $\lambda_p = \frac{h}{r}$

$$\lambda_p = \frac{h}{m_p v_0} \qquad \dots (1) \quad [:: m_d = 2m_p]$$
$$\lambda_d = \frac{h}{2m_p v_0} \qquad \dots (2)$$

Now, divided eqn. (1) by eqn. (2)

$$\frac{\lambda_p}{\lambda_d} = \frac{\frac{h}{m_p v_0}}{\frac{h}{2m_p v_0}} \quad \text{or} \quad \frac{\lambda_p}{\lambda_d} = \frac{2}{1}$$

62.
$$\lambda = \frac{h}{mv} \Rightarrow \frac{\lambda_p}{\lambda_e} = \frac{m_e}{m_p} = \frac{1}{1836}$$

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mE}} \quad \therefore \quad \lambda \propto \frac{1}{m} \quad \Rightarrow \quad \frac{\lambda_{\text{proton}}}{\lambda_{\text{alpha}}} = \sqrt{\frac{m_{\text{alpha}}}{m_{\text{proton}}}}$$

$$m_{\text{alpha}} = 4m_{\text{proton}}$$

$$\therefore \quad \frac{\lambda_{\text{proton}}}{\lambda_{\text{alpha}}} = \sqrt{\frac{m_{\text{alpha}}}{m_{\text{proton}}}} = \sqrt{\frac{4}{1}} = 2$$

So, correct option is (b).

57. (d):
$$\lambda = \frac{h}{p} \Rightarrow \lambda \propto \frac{1}{p}$$

the graph is rectangular hyperbola.

58. (a) de Broglie wavelength is given by :

$$h = -\frac{h}{2}$$

$$\lambda = \frac{n}{mv}$$

(i) When velocity decreases, wavelength increases

(ii)
$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV_0}}; \quad \lambda \propto \frac{1}{\sqrt{V_0}}$$

when accelerating potential increases, the de-broglie wavelength decreases.

59. (c): By momentum of conservation

$$MV = m_1 v_1 + m_2 v_2 + m_3 v_4$$

Particle is at rest initially
$$\therefore m_1v_1 = -m_2v_2$$

 $|P_1| = |P_2|$

According to de-Broglie wavelength $\lambda = \frac{h}{n}$

$$\frac{\lambda_1}{\lambda_2} = \frac{P_2}{P_1} = 1$$

Shortcut 🕗

 In such question we can directly use the value hc = 1240 nm-eV instead of putting absolute values of h and c.

(b) For hydrogen atom, momentum, p = 2 × 10⁻²⁷ kg m s⁻¹

Speed,
$$v = \frac{p}{m} = \frac{2 \times 10^{-27}}{1.6 \times 10^{-27}} = 1.24 \,\mathrm{m \, s^{-1}}$$

67. de-Broglie wavelength of a charged particle accelerated through a potential difference V is given by,

$$\lambda = \frac{n}{\sqrt{2mqV}}$$

 \therefore For proton and α particle charges are q and 2q respectively,

$$\therefore \quad \lambda_p = \frac{h}{\sqrt{2m_p q V}} \qquad \dots (i)$$

From eqn (i) and (ii)



Key Points 🔇

The de-Broglie wavelength is independent of the nature of material particle and charge on it.

63. (b):
$$p = \frac{h}{\lambda} \Rightarrow$$
 momentum, $p \propto \frac{1}{\lambda}$
64. de-Broglie wavelength
 $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mKE}}$

 $\Rightarrow \lambda^2 KE = constant.$

65. According to Louis de-Broglie, wave is associated with every moving particle. These are called matter waves. ∵ de-Broglie wavelength of a charged particle accelerated

through a potential difference V is given by, $\lambda = \frac{h}{\sqrt{2mqV}}$

... For proton and α particle charges are q and 2q respectively,

$$\lambda_{p} = \frac{h}{\sqrt{2m_{p}qV}} \qquad ...(i)$$

$$\lambda_{\alpha} = \frac{h}{\sqrt{4m_{\alpha}qV}} \qquad ...(ii)$$

From eqn (i) and (ii)

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \left(\frac{4m_\alpha}{2m_p}\right)^{1/2} = 2\sqrt{2} \qquad (\text{As } m_\alpha = 4 m_p)$$

(a) Wavelength , λ = 331.5 nm

Energy,
$$E = \frac{hc}{\lambda} = \frac{1240 \text{ nm} - \text{eV}}{331.5 \text{ nm}} = 3.74 \text{ eV}$$

Momentum, $p = \frac{E}{c} = \frac{3.74 \times 1.6 \times 10^{-19}}{3 \times 10^8} = 2 \times 10^{-27} \text{ kg m s}^{-1}$

(ii) Again from eqn. (i)

$$\lambda = \frac{h}{p} = \frac{h}{mv} \therefore \frac{\lambda_p}{\lambda_\alpha} = \frac{m_\alpha v_\alpha}{m_p v_p}; 1 = \frac{4mv_\alpha}{mv_p} \text{ or } \frac{v_p}{v_\alpha} = \frac{4}{1}$$
$$v_p: v_\alpha = 4:1$$

 For same accelerating potential, a proton and a deutron have same kinetic energy.

(a) de Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2m(qV)}}$$
. So, $\lambda \propto \frac{1}{\sqrt{m}}$

Mass of a deutron is more than that of a proton. So, proton will have greater value of de-Broglie wavelength.

(b) Momentum, $p = \sqrt{2mK}$; $p \propto \sqrt{m}$

Mass of a deutron is more than that of a proton. So, a proton has less momentum.

Concept Applied

Deuteron consists of a proton and neutron, thus have

$$\frac{\lambda_{\alpha}}{\lambda_{p}} = \left(\frac{2m_{p}}{4m_{\alpha}}\right)^{1/2} = \frac{1}{2\sqrt{2}} \qquad (\text{As } m_{\alpha} = 4 m_{p})$$

Answer Tips 🥖

KE

- Charge on an α-particle is two times the charge on proton. Mass of an α-particle is four times the mass of a proton.
- 68. Given that λ is the wavelength of the photon. The

de-Broglie wavelength of the electron is $\lambda = \frac{h}{d}$.

Kinetic energy of electron,

$$E_{e} = \frac{1}{2}mv^{2} = \frac{1}{2}m\left(\frac{h}{m\lambda}\right)^{2} = \frac{h^{2}}{2m\lambda^{2}} \qquad ...(i)$$

We know that energy of photon is $E_p = \frac{nc}{\lambda}$...(ii) Dividing (i) by (ii),

$$\frac{E_e}{E_p} = \frac{h^2}{2m\lambda^2} \times \frac{\lambda}{hc}; E_p = \frac{2\lambda mc}{h} E_e$$

69. De-Broglie wavelength of a particle of mass *m* and charge *q* accelerating through a potential V is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}} \qquad \dots (i)$$

(i) Here, $m_p = m$, $q_p = e$, $m_\alpha = 4m_p = 4m$, $q_\alpha = 2q_p = 2e$ From eqn. (i)

$$\therefore \frac{\lambda_{p}}{\lambda_{\alpha}} = \sqrt{\frac{m_{\alpha}q_{\alpha}V_{\alpha}}{m_{p}q_{p}V_{p}}}$$

$$1 = \sqrt{\frac{4m \times 2e \times V_{\alpha}}{m \times e \times V_{p}}}$$

$$\therefore \frac{V_{p}}{V_{\alpha}} = \frac{8}{1}; V_{p}: V_{\alpha} = 8:1$$

Energy of light of wavelength 430 nm (E_1) = $\frac{1240 \text{ eV nm}}{430}$ E_1 = 2.88 eV

As it is for wavelength of 450 nm (E_2) = $\frac{1240 \text{ eV nm}}{450}$ E_2 = 2.76 eV

For wavelength of 660 nm $(E_3) = \frac{1240 \text{ eV nm}}{660}$

We have seen here clearly that E_1 and E_2 have energy more than required energy for remove photoelectron from surface.

So, in both cases photoemission take place.

73. de-Broglie wavelength of a particle of mass *m* and charge *q* accelerating through a potential V is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}} \qquad \dots (i)$$

Here, $m_p = m$, $q_p = e$, $m_\alpha = 4m_p = 4m$, $q_\alpha = 2q_p = 2e$ From eqn. (i)

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According to Einstein's photoelectric effect

$$E = W + \frac{1}{2}mv^2$$

Since work function of the surface is negligible, the above equation becomes

 $E = \frac{1}{2}mv^{2}$ $mv = \sqrt{2mE}$ If λ is de-Broglie wavelength of the emitted electrons, then $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$ $K_{v} \longrightarrow$

72. (a) Let, a proton and an α-particle are moving with speed = υ

The mass of proton $m_p = m$

The mass of α -particle $m_{\text{He}} = 4m$

De-Broglie wavelength for proton, $\lambda_1 = \frac{h}{m_0 v}$

$$\lambda_1 = \frac{h}{mu}$$

De-Broglie wavelenth for α -particle, $\lambda_2 = \frac{h}{m_{\rm b}}$

$$\lambda_2 = \frac{h}{4mv}$$

So, we can see clearly that $\lambda_1 > \lambda_2$.

Therefore, proton has greater value of De-broglie wavelength due to its less mass.

(b) There are three lights of wavelength 430 nm, 450 nm and 660 nm and the threshold wavelength of metal is 600 nm.

As we know that $E = \frac{hc}{\lambda}$

:: hc = 1240 eV nm

So threshold energy for remove a photoelectron from metal surface (E₀)

$$E_0 = \frac{1240 \text{ eV nm}}{600 \text{ nm}}$$
; $E_0 = 2.07 \text{ eV}$

uses yellow light of wavelength (5700 Å to 5900 Å). Hence, the resolving power of an electron microscope is much greater than that of optical microscope.

CBSE Sample Questions

 (b) : Photoelectric emission from a given surface of metal can take place when its work function is less than the energy of incident photon. (1)

2. (c): Work function for a metal surface,
$$\phi = \frac{hc}{\lambda_0}$$

Given that, $\phi = 4.14 \text{ eV} = 4.14 \times 1.6 \times 10^{-19} \text{ J}$
 $\therefore \quad \lambda_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.14 \times 1.6 \times 10^{-19}} = 3000 \text{ Å}$ (1)

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}}; \ 1 = \sqrt{\frac{4m \times 2e \times V_\alpha}{m \times e \times V_p}} \qquad (\because \lambda_p = \lambda_\alpha)$$
$$\therefore \quad \frac{V_p}{V_\alpha} = \frac{8}{1}; \ V_p : V_\alpha = 8:1$$

74. (a) (i) For same accelerating potential, a proton and an electron have same kinetic energy. The de-broglie wavelength associated with same potential V is given by,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2m(qV)}}$$

So, $\lambda = \frac{1}{\sqrt{m}}$

As electron's mass is lesser than proton. Thus $\lambda_e > \lambda_p$.

(ii) Momentum, $p = \sqrt{2mK}$ or $p \propto \sqrt{m}$

As electron's mass is lesser than proton. Thus momentum of electron is lesser than proton.

(b) De-Broglie wavelength of a particle

$$\lambda = \frac{h}{n}$$
 or $\lambda p = h = \text{constant}$

It shows a rectangular hyperbola.

75.
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$
 or $\lambda = \frac{12.27}{\sqrt{V}}$ Å
∴ $\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{(2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 50 \times 10^3)}}$
= 5.49 × 10⁻¹² m

The resolving power of an electron microscope is much better than that of optical microscope.

Resolving power of a microscope is inversely proportional to the wavelength of light.

This suggests that to improve resolution, we have to use shorter wavelength.

For an electron microscope, the electrons are accelerated through a 50,000 V potential difference. Thus the wavelength of electrons is found to be 10⁻¹² m.

As, λ is very small (approximately 10⁻⁵ times smaller) for electron microscope than an optical microscope which

 (a): Electrons being emitted as photoelectrons have different velocities. As all the electrons do not occupy the same level of energy but they occupy continuous band and levels. So, electrons ejected out from different levels come out with different energies. (1)

Given, stopping potential, V_s = 4V (1)
 K.E._{max} = eV_s

$$K.E_{max} = e \times 4 = = 4 eV$$

7. Given,
$$\lambda = 2000 \text{ Å} = 2000 \times 10^{-10} \text{ m}$$

 $W_0 = 4.2 \text{ eV}$
 $h = 6.63 \times 10^{-34} \text{ J s}$
(a) $\frac{hc}{\lambda} = W_0 + \text{K.E.}$ or $\text{K.E.} = \frac{hc}{\lambda} - W_0$ (1)

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 (a) The variable X on the horizontal axis is collector plate potential.

The point A on the horizontal axis represents stopping potential. (1)



Variation of photoelectric current with collector plate potential for different frequencies of incident radiation. (c)



 Retarding potential Collector plate potential —>
 Variation of photocurrent with collector plate potential for different intensities of incident radiation. (1)

4. Main implications:

 Kinetic energy of emitted electrons depends upon frequency, but not on intensity of radiation.

 (ii) There exist a frequency of radiation below which no photoemission takes place, how high intensify of radiation may be known as threshold frequency.
 (2)

Explanation of wave nature of radiation fails to explain photoelectric effect.

According to wave theory, when light falls on a metal surface, energy is continuously distributed over the surface. All electrons may be ejected only when it acquires energy more than the work function. So, if we use low intensity source, it should take hours for photoelectric emission, but photoelectric effect is almost a spontaneous process.

$$=\frac{(6.63\times10^{-34})\times(3\times10^{8})}{(2000\times10^{-10})}\times\frac{1}{1.6\times10^{-19}}\text{eV}-4.2\,\text{eV}$$

(b) The energy of the emitted electrons does not depend upon intensity of incident light, hence the energy remains unchanged. (1)

(c) For this surface, electrons will not be emitted as the energy of incident light (6.2 eV) is less than the work function (6.5 eV) of the surface. (1)

 From the observations made (parts A and B) on the basis of Einstein's photoelectric equation, we can draw following conclusions:

- For surface A, the threshold frequency is more than 10¹⁵ Hz, hence no photoemission is possible.
- For surface B, the threshold frequency is equal to the frequency of given radiation. Thus, photoemission takes place but kinetic energy of photoelectrons is zero.
- For surface C, the threshold frequency is less than 10¹⁵ Hz. So, photoemission occurs and photoelectrons have some kinetic energy. (3)

 (a) De-broglie reasoned out that nature was symmetrical and two basic physical entities - mass and radiation must be symmetrical. If radiation shows dual aspect than matter should do so.
 (1)

de-broglie equation,
$$\lambda = \frac{n}{p}$$

For photon,
$$p = \frac{hv}{m}$$

Therefore, $\frac{h}{p} = \frac{c}{v} = \lambda$, wavelength of electromagnetic radiation.

(b) As
$$\lambda = \frac{h}{\sqrt{2mK}}$$

So, alpha particle will be having shortest De-Broglie wavelength compared to deutrons. (2) k = qV

$$\begin{split} \frac{\lambda_d}{\lambda_\alpha} &= \frac{\sqrt{m_\alpha q_\alpha V}}{\sqrt{m_d q_d V}} = \sqrt{\frac{2m_d \times 2q_d}{m_d q_d}} & \begin{pmatrix} m_\alpha = 2m_d \\ q_\alpha = 2q_d \end{pmatrix} \\ \frac{\lambda_d}{\lambda_\alpha} &= \frac{2}{1} \implies \lambda_d : \lambda_\alpha = 2:1 \end{split}$$

